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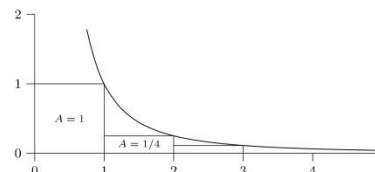
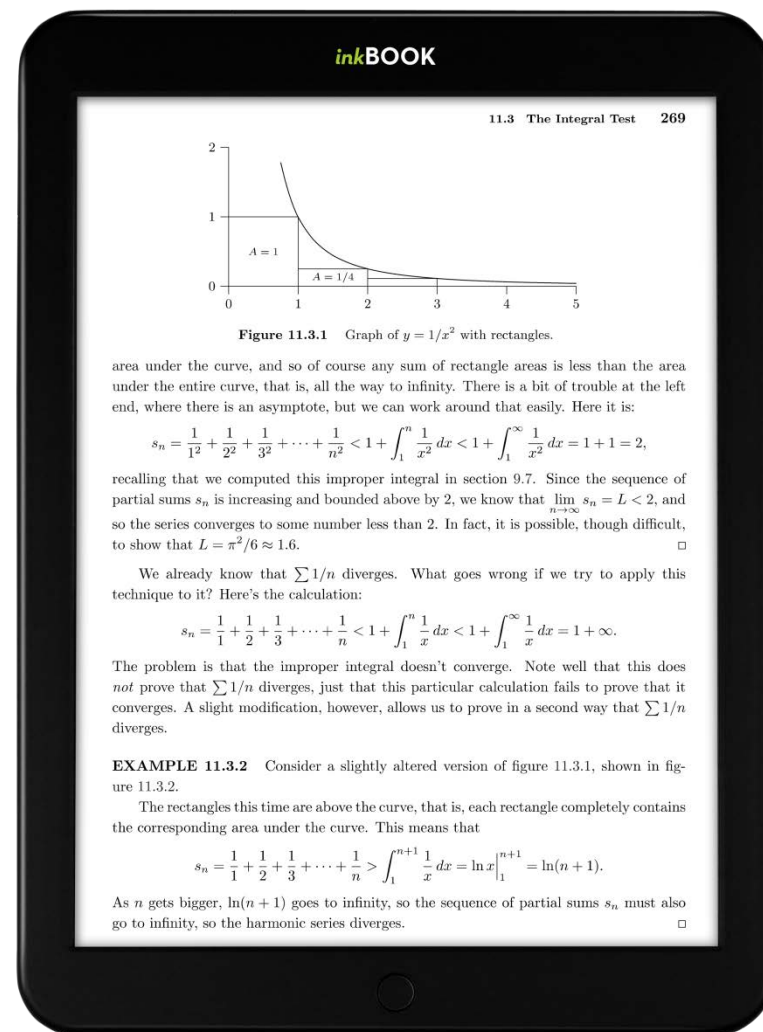


Figure 11.3.1 Graph of $y = 1/x^2$ with rectangles.

area under the curve, and so of course any sum of rectangle areas is less than the area under the entire curve, that is, all the way to infinity. There is a bit of trouble at the left end, where there is an asymptote, but we can work around that easily. Here it is:

$$s_n = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} < 1 + \int_1^n \frac{1}{x^2} dx < 1 + \int_1^\infty \frac{1}{x^2} dx = 1 + 1 = 2,$$

recalling that we computed this improper integral in section 9.7. Since the sequence of partial sums s_n is increasing and bounded above by 2, we know that $\lim_{n \rightarrow \infty} s_n = L < 2$, and so the series converges to some number less than 2. In fact, it is possible, though difficult, to show that $L = \pi^2/6 \approx 1.6$. \square

We already know that $\sum 1/n$ diverges. What goes wrong if we try to apply this technique to it? Here's the calculation:

$$s_n = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} < 1 + \int_1^n \frac{1}{x} dx < 1 + \int_1^\infty \frac{1}{x} dx = 1 + \infty.$$

The problem is that the improper integral doesn't converge. Note well that this does *not* prove that $\sum 1/n$ diverges, just that this particular calculation fails to prove that it converges. A slight modification, however, allows us to prove in a second way that $\sum 1/n$ diverges.

EXAMPLE 11.3.2 Consider a slightly altered version of figure 11.3.1, shown in figure 11.3.2.

The rectangles this time are above the curve, that is, each rectangle completely contains the corresponding area under the curve. This means that

$$s_n = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} > \int_1^{n+1} \frac{1}{x} dx = \ln x \Big|_1^{n+1} = \ln(n+1).$$

As n gets bigger, $\ln(n+1)$ goes to infinity, so the sequence of partial sums s_n must also go to infinity, so the harmonic series diverges. \square